

marks in the level above. Teachers should award the lower marks if the student's work demonstrates the qualities described to a lesser extent; the work may be close to achieving marks in the level below.

Only whole numbers should be recorded; partial marks, (fractions or decimals) are not acceptable.

Teachers should not think in terms of a pass or fail boundary, but should concentrate on identifying the appropriate descriptor for each assessment criterion.

The highest-level descriptors do not imply faultless performance but should be achievable by a student. Teachers should not hesitate to use the extremes if they are appropriate descriptions of the work being assessed.

A student who attains a high achievement level in relation to one criterion will not necessarily attain high achievement levels in relation to the other criteria. Similarly, a student who attains a low achievement level for one criterion will not necessarily attain low achievement levels for the other criteria. Teachers should not assume that the overall assessment of the students will produce any particular distribution of marks.

It is recommended that the assessment criteria be made available to students.

Internal assessment details

Mathematical exploration

Duration: 10 to 15 hours

Weighting: 20%

Introduction

The internally-assessed component in this course is a mathematical exploration. This is a short report written by the student based on a topic chosen by him or her, and it should focus on the mathematics of that particular area. The emphasis is on mathematical communication (including formulae, diagrams, graphs, tables and so on), with his or her own focus, with the teacher providing feedback via, for example, discussion and interview. This will allow the students to develop areas of interest to them without a time constraint as in an examination, and allow all students to experience a feeling of success.

The final report should be approximately 12-20 pages long with double line spacing. It can be either word processed or handwritten. Students should be able to explain all stages of their work in such a way that demonstrates clear understanding. While there is no requirement that students present their work in class, it should be written in such a way that their peers would be able to follow it fairly easily. The report should include a detailed bibliography, and sources need to be referenced in line with the IB academic honesty policy. Direct quotes must be acknowledged.

The purpose of the exploration

The aims of the Mathematics: analysis and approaches and Mathematics: applications and interpretation courses at both SL and HL are carried through into the objectives that are formally assessed as part of the course, through either written examination papers or the exploration, or both. In addition to testing the objectives of the course, the exploration is intended to provide students with opportunities to increase their understanding of mathematical concepts and processes, and to develop a wider appreciation of mathematics. These are noted in the aims of the course. It is intended that, by doing the exploration, students benefit from the mathematical activities undertaken and find them both stimulating and rewarding. It will enable students to acquire the attributes of the IB learner profile.

The specific purposes of the exploration are to:

- develop students' personal insight into the nature of mathematics and to develop their ability to ask their own questions about mathematics
- provide opportunities for students to complete a piece of mathematical work over an extended period of time
- enable students to experience the satisfaction of applying mathematical processes independently

- provide students with the opportunity to experience for themselves the beauty, power and usefulness of mathematics
- encourage students, where appropriate, to discover, use and appreciate the power of technology as a mathematical tool
- enable students to develop the qualities of patience and persistence, and to reflect on the significance of their work
- provide opportunities for students to show, with confidence, how they have developed mathematically.

Management of the exploration

Work on the exploration should be incorporated into the course so that students are given the opportunity to learn the skills needed. Time in class can therefore be used for general discussion of areas of study, as well as familiarizing students with the criteria. Further details on the development of the exploration are included in the teacher support material.

Internal assessment criteria—SL and HL

The exploration is internally assessed by the teacher and externally moderated by the IB using assessment criteria that relate to the objectives for mathematics.

Each exploration is assessed against the following five criteria. The final mark for each exploration is the sum of the scores for each criterion. The maximum possible final mark is 20.

Students will not receive a grade for their mathematics course if they have not submitted an exploration.

Criterion A	Presentation
Criterion B	Mathematical communication
Criterion C	Personal engagement
Criterion D	Reflection
Criterion E	Use of mathematics

Criterion A: Presentation

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration has some coherence or some organization.
2	The exploration has some coherence and shows some organization.
3	The exploration is coherent and well organized.
4	The exploration is coherent, well organized, and concise.

The “presentation” criterion assesses the organization and coherence of the exploration.

A **coherent** exploration is logically developed, easy to follow and meets its aim. This refers to the overall structure or framework, including introduction, body, conclusion and how well the different parts link to each other.

A **well-organized** exploration includes an introduction, describes the aim of the exploration and has a conclusion. Relevant graphs, tables and diagrams should accompany the work in the appropriate place and not be attached as appendices to the document. Appendices should be used to include information on large data sets, additional graphs, diagrams and tables.

A **concise** exploration does not show irrelevant or unnecessary repetitive calculations, graphs or descriptions.

The use of technology is not required but encouraged where appropriate. However, the use of analytic approaches rather than technological ones does not necessarily mean lack of conciseness, and should not be penalized. This does not mean that repetitive calculations are condoned.

Criterion B: Mathematical communication

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration contains some relevant mathematical communication which is partially appropriate.
2	The exploration contains some relevant appropriate mathematical communication.
3	The mathematical communication is relevant, appropriate and is mostly consistent.
4	The mathematical communication is relevant, appropriate and consistent throughout.

The “mathematical communication” criterion assesses to what extent the student has:

- used appropriate mathematical language (**notation, symbols, terminology**). Calculator and computer notation is acceptable only if it is software generated. Otherwise it is expected that students use appropriate mathematical notation in their work
- defined **key terms** and variables, where required
- used **multiple forms of mathematical representation**, such as formulae, diagrams, tables, charts, graphs and models, where appropriate
- used a **deductive method** and set out proofs logically where appropriate

Examples of level 1 can include graphs not being labelled, consistent use of computer notation with no other forms of correct mathematical communication.

Level 4 can be achieved by using only one form of mathematical representation as long as this is appropriate to the topic being explored. For level 4, any *minor* errors that do not impair clear communication should not be penalized.

Criterion C: Personal engagement

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of some personal engagement.
2	There is evidence of significant personal engagement.
3	There is evidence of outstanding personal engagement.

The “personal engagement” criterion assesses the extent to which the student engages with the topic by exploring the mathematics and making it their own. It is not a measure of effort.

Personal engagement may be recognized in different ways. These include thinking independently or creatively, presenting mathematical ideas in their own way, exploring the topic from different perspectives, making and testing predictions. Further (but not exhaustive) examples of personal engagement at different levels are given in the teacher support material (TSM).

There must be evidence of personal engagement demonstrated in the student's work. It is not sufficient that a teacher comments that a student was highly engaged.

Textbook style explorations or reproduction of readily available mathematics without the candidate's own perspective are unlikely to achieve the higher levels.

Significant: The student demonstrates authentic personal engagement in the exploration on a few occasions and it is evident that these drive the exploration forward and help the reader to better understand the writer's intentions.

Outstanding: The student demonstrates authentic personal engagement in the exploration in numerous instances and they are of a high quality. It is evident that these drive the exploration forward in a creative way. It leaves the impression that the student has developed, through their approach, a complete understanding of the context of the exploration topic and the reader better understands the writer's intentions.

Criterion D: Reflection

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited reflection.
2	There is evidence of meaningful reflection.
3	There is substantial evidence of critical reflection.

The "reflection" criterion assesses how the student reviews, analyses and evaluates the exploration. Although reflection may be seen in the conclusion to the exploration, it may also be found throughout the exploration.

Simply describing results represents **limited reflection**. Further consideration is required to achieve the higher levels.

Some ways of showing **meaningful reflection** are: linking to the aims of the exploration, commenting on what they have learned, considering some limitation or comparing different mathematical approaches.

Critical reflection is reflection that is crucial, deciding or deeply insightful. It will often develop the exploration by addressing the mathematical results and their impact on the student's understanding of the topic. Some ways of showing critical reflection are: considering what next, discussing implications of results, discussing strengths and weaknesses of approaches, and considering different perspectives.

Substantial evidence means that the critical reflection is present throughout the exploration. If it appears at the end of the exploration it must be of high quality and demonstrate how it developed the exploration in order to achieve a level 3.

Further (but not exhaustive) examples of reflection at different levels are given in the teacher support material (TSM).

Criterion E: Use of mathematics—SL

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used.
2	Some relevant mathematics is used. Limited understanding is demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. Limited understanding is demonstrated.

Achievement level	Descriptor
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is partially correct. Some knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is mostly correct. Good knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Thorough knowledge and understanding are demonstrated.

The “Use of mathematics” SL criterion assesses to what extent students use mathematics that is **relevant** to the exploration.

Relevant refers to mathematics that supports the development of the exploration towards the completion of its aim. Overly complicated mathematics where simple mathematics would suffice is not relevant.

Students are expected to produce work that is **commensurate with the level** of the course, which means it should not be completely based on mathematics listed in the prior learning. The mathematics explored should either be part of the syllabus, or at a similar level.

A key word in the descriptor is **demonstrated**. The command term demonstrate means “to make clear by reasoning or evidence, illustrating with examples or practical application”. Obtaining the correct answer is not sufficient to demonstrate understanding (even some understanding) in order to achieve level 2 or higher.

For knowledge and understanding to be **thorough** it must be demonstrated throughout.

The mathematics can be regarded as **correct** even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome.

Students are encouraged to use technology to obtain results where appropriate, but **understanding must be demonstrated** in order for the student to achieve higher than level 1, for example merely substituting values into a formula does not necessarily demonstrate understanding of the results.

The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single topic (or sub-topic) from the syllabus. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with the level of the course and understood by the student, then it can achieve a high level in this criterion.

Criterion E: Use of mathematics—HL

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used. Limited understanding is demonstrated.
2	Some relevant mathematics is used. The mathematics explored is partially correct. Some knowledge and understanding is demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Some knowledge and understanding are demonstrated.

Achievement level	Descriptor
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Good knowledge and understanding are demonstrated.
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct and demonstrates sophistication or rigour. Thorough knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is precise and demonstrates sophistication and rigour. Thorough knowledge and understanding are demonstrated.

The “Use of mathematics” HL criterion assesses to what extent students use **relevant** mathematics in the exploration.

Students are expected to produce work that is **commensurate with the level** of the course, which means it should not be completely based on mathematics listed in the prior learning. The mathematics explored should either be part of the syllabus, at a similar level or slightly beyond. However, mathematics of a level slightly beyond the syllabus is **not** required to achieve the highest levels.

A key word in the descriptor is **demonstrated**. The command term demonstrate means to make clear by reasoning or evidence, illustrating with examples or practical application. Obtaining the correct answer is not sufficient to demonstrate understanding (even some understanding) in order to achieve level 2 or higher.

For knowledge and understanding to be thorough it must be demonstrated throughout. Lines of reasoning must be shown to justify steps in the mathematical development of the exploration.

Relevant refers to mathematics that supports the development of the exploration towards the completion of its aim. Overly complicated mathematics where simple mathematics would suffice is not relevant.

The mathematics can be regarded as **correct** even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome. **Precise** mathematics is error-free and uses an appropriate level of accuracy at all times.

Sophistication: To be considered as sophisticated the mathematics used should be commensurate with the HL syllabus or, if contained in the SL syllabus, the mathematics has been used in a complex way that is beyond what could reasonably be expected of an SL student. Sophistication in mathematics may include understanding and using challenging mathematical concepts, looking at a problem from different perspectives and seeing underlying structures to link different areas of mathematics.

Rigour involves clarity of logic and language when making mathematical arguments and calculations. Mathematical claims relevant to the development of the exploration must be justified or proven.

Students are encouraged to use technology to obtain results where appropriate, but **understanding must be demonstrated** in order for the student to achieve level 1 or higher, for example merely substituting values into a formula does not necessarily demonstrate understanding of the results.

The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single topic (or sub-topic) from the syllabus. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with the level of the course and understood by the student, then it can achieve a high level in this criterion.

Glossary of command terms

Command terms for Mathematics: applications and interpretation

Students should be familiar with the following key terms and phrases used in examination questions, which are to be understood as described below. Although these terms will be used frequently in examination questions, other terms may be used to direct students to present an argument in a specific way.

Command term	Definition
Calculate	Obtain a numerical answer showing the relevant stages in the working.
Comment	Give a judgment based on a given statement or result of a calculation.
Compare	Give an account of the similarities between two (or more) items or situations, referring to both (all) of them throughout.
Compare and contrast	Give an account of similarities and differences between two (or more) items or situations, referring to both (all) of them throughout.
Construct	Display information in a diagrammatic or logical form.
Contrast	Give an account of the differences between two (or more) items or situations, referring to both (all) of them throughout.
Deduce	Reach a conclusion from the information given.
Demonstrate	Make clear by reasoning or evidence, illustrating with examples or practical application.
Describe	Give a detailed account.
Determine	Obtain the only possible answer.
Differentiate	Obtain the derivative of a function.
Distinguish	Make clear the differences between two or more concepts or items.
Draw	Represent by means of a labelled, accurate diagram or graph, using a pencil. A ruler (straight edge) should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted (if appropriate) and joined in a straight line or smooth curve.
Estimate	Obtain an approximate value.
Explain	Give a detailed account including reasons or causes.
Find	Obtain an answer showing relevant stages in the working.
Hence	Use the preceding work to obtain the required result.
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.
Identify	Provide an answer from a number of possibilities.
Integrate	Obtain the integral of a function.

Command term	Definition
Interpret	Use knowledge and understanding to recognize trends and draw conclusions from given information.
Investigate	Observe, study, or make a detailed and systematic examination, in order to establish facts and reach new conclusions.
Justify	Give valid reasons or evidence to support an answer or conclusion.
Label	Add labels to a diagram.
List	Give a sequence of brief answers with no explanation.
Plot	Mark the position of points on a diagram.
Predict	Give an expected result.
Prove	Use a sequence of logical steps to obtain the required result in a formal way.
Show	Give the steps in a calculation or derivation.
Show that	Obtain the required result (possibly using information given) without the formality of proof. "Show that" questions do not generally require the use of a calculator.
Sketch	Represent by means of a diagram or graph (labelled as appropriate). The sketch should give a general idea of the required shape or relationship, and should include relevant features.
Solve	Obtain the answer(s) using algebraic and/or numerical and/or graphical methods.
State	Give a specific name, value or other brief answer without explanation or calculation.
Suggest	Propose a solution, hypothesis or other possible answer.
Verify	Provide evidence that validates the result.
Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.